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Mechanisms with No Regret: Welfare Economics and Information Reconsidered

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MECHANISMS WITH NO REGRET:

Welfare Economics and Information Reconsidered

by

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Abstract

This paper achieves four objectives relating to the design of efficient mechanisms in economies with asymmetric information. First, we show that it is impossible to design an individually rational and efficient mechanism using Bayesian equilibrium as the solution concept. Under complete information, however, such mechanisms can be designed. Second, we attempt to bridge this gap between complete information economies and an important sub-class of asymmetric information economies by exploring the possibilities with "mechanisms with no regret" that leak information. A complete characterization is given of "No regret-implementation" using such mechanisms. These mechanisms are played in four stages including a "regret phase" and yield a refinement of Green and Laffont's (1987) *posterior implementability* concept. Third, it is shown that though such mechanisms cannot implement *interim* individually rational and efficient performance standards, they can implement *ex post* individually rational and efficient standards. Finally, it is shown that even under asymmetric information such mechanisms implement *any* Nash implementable performance standard, such as the core or the Walrasian correspondence.

Key Words: *Implementation, Bayesian equilibrium with no regret, individual rationality-efficiency, asymmetric information.* **JEL Code:** *026.*

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1. Introduction

In economies with asymmetric information, a *performance standard* is a non-empty set of socially "desirable" state-contingent allocations. Given the incompleteness of information about the realized state of the world, a *mechanism* is required to *implement* a given performance standard. An implementing mechanism is a game of incomplete information with a non-empty set of equilibrium outcomes that is contained within the standard. (*Full implementation* requires coincidence of the two sets.) Most economies aspire towards the two basic objectives of individual rationality and efficiency. This paper reports a fundamental difficulty with any attempt to design mechanisms for implementing individually rational-efficient performance standards, and then proposes conditions under which there exists a solution to this crucial problem. In light of the difficulties that were just alluded to, this is the only result that demonstrates the possibility of designing individually rational-efficient mechanisms in a wide class of asymmetric information economies. Moreover, we show that despite the information asymmetry it is possible to implement any standard that is Nash-implementable (i.e. with complete information), such as the core or the Walrasian correspondence.

By the Revelation Principle (Myerson (1979), Dasgupta, Hammond and Maskin (1979), Harris and Townsend (1981), etc.), any performance standard that has a non-empty intersection with the set of Bayesian equilibrium outcomes of a game must satisfy an incentive comaptibility (or self-selection) condition. We know of no satisfactory performance standard that meets this condition. Thus, if we are interested in questions of mechanism design in a wide class of environments, the conclusions are

negative right away.

To surmount this initial hurdle, Palfrey and Srivastava (1987a) invoke Postlewaite and Schmeidler's (1986) restriction on the informational structure -- *non-exclusivity of information (NEI)* -- and then ask whether performance standards that were fully implementable under complete information can be fully implemented when information is asymmetric. The NEI restriction essentially requires that if all agents except one pool their private information they can deduce the information of the remaining agent. It ensures incentive compatibility but is clearly restrictive. Unfortunately, some restriction of this nature is necessary to evaluate the implementability of natural economic performance standards. The striking message of Palfrey and Srivastava is: despite the restriction and despite the fact that incentive compatibility is satisfied, full Bayesian implementation of either the interim or the ex post efficiency standard (as defined in Holmström and Myerson (1983)) is impossible. This is in sharp contrast to the positive results obtained under complete information.

Palfrey and Srivastava's (1987a) negative results provide the starting point for this paper. We extend their arguments and show that the situation is even more grim than is implied by their results. Define an *efficient Bayesian mechanism* as a game such that all of its Bayesian equilibrium outcomes are efficient (in either an interim or ex post sense). However disappointing the Palfrey and Srivastava conclusions may be, they do not imply that we can never hope to design an efficient mechanism since there still exists the possibility of Bayesian-implementing (as opposed to fully Bayesian-implementing) the efficiency standard or one of its subsets.

The first part of this paper argues that, indeed, even under NEI, we cannot hope to find an efficient Bayesian mechanism if we impose an

additional restriction of individual rationality on the standard. The latter rules out uninteresting outcomes that assign all resources to one individual. Such outcomes are efficient and trivially implementable. To summarize, both the interim and the ex post individual rationality-efficiency standard (and their respective subsets) are non-implementable in Bayesian equilibria. An additional implication of this part of the paper is: when one is interested in checking for impossibility results, "implementation" (as opposed to "full implementation") is the notion that one should focus on; a standard that is not fully implementable may be implementable whereas one that is not implementable can never be fully implementable.

These impossibility results indicate the cost, in terms of social welfare, that is imposed on society by the presence of information asymmetry. Under complete information, the literature on Nash-implementation, (Maskin (1977), Hurwicz, Maskin and Postlewaite (1984), etc.) has shown that individually rational-efficient mechanisms do indeed exist. Can this gap be filled by a mechanism that not only allocates resources but also "leaks" information?

The next step in the agenda is to tackle the problem of non-existence of individually rational-efficient Bayesian mechanisms by addressing the question posed in the previous paragraph. We take advantage of the fact that the mechanism designer has the flexibility to not only choose the game but also how it is played. Following the lead of Green and Laffont (1987), we study the case of mechanisms with a "regret" phase. Green and Laffont do not provide any specific structure to the changes in the rules of the game. In this paper, we adopt a particular sequence of moves that allows for a regret phase in a simultaneous-move game. This sequence is a slight

modification of the structure that is implicit in Green and Laffont's model. The resulting equilibrium that we obtain is a refinement of Bayesian equilibria and of *posterior implementable* equilibria (due to Green and Laffont).

The structure that we employ evolves in four stages and is informally described with the help of an example in the following paragraphs.

Example 1: Consider a game with two players and three states of the world: "Rain", "Shine" and "Clouds". Player 1 chooses T or B and is completely uninformed. Her prior beliefs are that there is an equal chance of any one of the states occurring. Player 2 chooses L, M or R and is completely informed. Their payoffs are given in Figure 1. This is the *original game*. The pure strategy Bayesian equilibria of the original game are given in Figure 2. A game designer can introduce a regret phase in rules of play of this game.

[Insert Figures 1 and 2 here.]

The game with a regret phase is played in four stages. In the first stage, the players and the game designer calculate the set of Bayesian equilibria in the original game. This constitutes the maximal set of possible outcomes. Next, Nature selects a state of the world. In the second stage, the players submit to the designer the actions they intend to take in every equilibrium. These submissions are not commitments and are made public knowledge by the designer. In the third stage, the agents are given the opportunity to entertain the desire to unilaterally deviate from one or more of the Bayesian equilibria. In the final stage, (as is usually assumed in game theory) by some unbiased method a particular Bayesian equilibrium, say $\hat{s}$, is selected from the set comprising equilibria from which no player wants to unilaterally deviate. $\hat{s}$ becomes common knowledge

among the players and the designer. The game designer distributes the outcome/payoffs to the players by checking the actions that each player had submitted for $\hat{s}$ and using the payoff matrices in Figure 1.

It can be observed that this structure is different from the one that underlies Green and Laffont's notion of posterior implementable equilibria. In their case, the unbiased selection of the equilibrium is done before the second stage. Thus, they would require that in the second stage the players submit the actions that they intend to take in the particular equilibrium that is chosen.

To return to the example, a Bayesian equilibrium will survive the third stage if and only if the information conveyed by the second stage public submissions is such that neither player has an incentive to unilaterally deviate in stage three from the action he/she had submitted for that equilibrium in the previous stage. Such an equilibrium is referred to as a *Bayesian equilibrium with no regret.*

Since equilibria are pairs of strategies that are common knowledge functions, Player 1 could conceivably acquire some information if Player 2's action were associated with a unique state in some Bayesian equilibrium. However, not all Bayesian equilibria convey information in this manner.

Let the set of Bayesian equilibria with no regret be denoted $E(\Gamma)$. Every Bayesian equilibrium with no regret must be immune to information revealed by any other Bayesian equilibrium with no regret. This follows from the fact that in stage four a Bayesian equilibrium with no regret will be chosen to determine the final outcome. Given that in stage two the players do not know which one will be chosen, they will not have an incentive to unilaterally make arbitrary submissions for any equilibrium in

$E(\Gamma)$. This rules out unilateral manipulation on the part of the players by the submission of messages that could mislead others. An equilibrium that is not in $E(\Gamma)$ does not convey information since the players may submit messages corresponding to such an equilibrium to mislead others about the state of the world knowing that such equilibria shall never be used to determine the final outcome.

The calculation of the set of Bayesian equilibria with no regret may appear somewhat circular. We shall show how it is determined for the example at hand.

Observe that the functions s', s'' and s^* carry information if the state is "Clouds". The equilibria s' and s^* remain self-enforcing even if the information were revealed to Player 1 in the "Clouds" state by s', s'' or s^*. This gives us a sufficiency condition: s' and s^* are Bayesian equilibria with no regret. Are there any others? A necessary condition is that a Bayesian equilibrium with no regret must not be destroyed by information conveyed by another Bayesian equilibrium with no regret. It can be checked that in s and s'' Player 1 has an incentive to deviate in the "Clouds" state given the information conveyed by s' and s^*. Thus, s' and s^* constitute the set of Bayesian equilibria with no regret, $E(\Gamma)$.

We conclude this example with two observations. First, note that the Bayesian equilibria with no regret (e.g. s^*) are not necessarily complete information Nash equilibria of the game. Second, if we had employed Green and Laffont's criterion, s, s' and s^* would have survived and s'' would have been eliminated. In sum, our criterion of regret requires greater robustness from an equilibrium. While this is a strength from the viewpoint of stability, it makes it more difficult to guarantee existence of such equilibria. In this paper, we shall, however, prove the existence

of the set of Bayesian equilibria with no regret for any mechanism that we use for our implementation result.

The positive results in the paper make use of mechanisms with a regret phase as outlined in the example above. A complete characterization is provided for performance standards that are *No-regret-(NR)-implementable* (i.e. we use Bayesian equilibrium with no regret as the appropriate equilibrium concept). The characterization results are used to prove the following arguments. The interim individual rationality-efficiency standard cannot be NR-implemented. However, the ex post individual rationality-efficiency standard is NR-implementable. In general, any standard that is Nash-implementable under complete information is also NR-implementable under asymmetric information. The sub-class of environments in which we demonstrate these results satisfy the NEI condition. Thus, we are successful in bridging the gap (in terms of efficient mechanism design) between NEI environments and complete information environments.

As mentioned earlier, NEI is clearly restrictive. However, given the constraints imposed by the Revelation Principle, some condition "close" to NEI is also necessary to obtain any positive result (Blume and Easley (1987) prove the necessity of NEI for implementability of the Rational Expectations Equilibrium standard). Moreover, NEI is satisfied in a large variety of environments involving informational asymmetry. A few of these are listed below:

(i) In large enough economies the private information of any single individual can be fairly accurately predicted by pooling the information observed by the rest of the population.

(ii) Individuals can be one of several "types". The proportions of

each type in the population is known.

(iii) Information may depend on physical possession of some commodity whose different varieties and total quantity is known. Each individual's private holding is not common knowledge.

(iv) There are at least two fully informed individuals in the economy, e.g. in certain markets with several sellers and buyers, all sellers are presumed to be informed and all buyers are uninformed.

(v) Each individual has at least one other person who can verify his/her information, e.g. close relatives, witnesses, etc.

(vi) Information is held by coalitions of individuals with the minimum size of each coalition being two, e.g. family units with common preferences over goods, preferences are determined by the school one attends, collaborators who know each other's information, etc.

In sum, given that most individuals exist within certain social institutions, truly exclusive relevant information may indeed be a rarity. The success of real-world information collection mechanisms such as courts, inquiry commissions, etc., which rely on gathering enough individuals to be able to deduce the private information held by one individual, indicates that NEI environments constitute a significant class. Thus, for a vast number of realistic applications, a positive result that relies on the NEI condition represents a significant improvement over impossibility results or positive results under complete information.

Juxtaposed with related literature, our findings have several interesting implications: (i) Green and Laffont's (1987) conclusions regarding the application of games with a regret phase to the problem of mechanism design were largely negative. We employ a stronger definition of implementability and equilibrium and show that such games do, indeed, have

important applications towards deriving positive results. (ii) The most widely studied concept of implementation is Nash-implementation whose origins lie in the classic work of Maskin (1977). Maskin's characterization of Nash-implementable standards (also see Saijo (1988)) has been criticized on the grounds that it applies only to complete information settings. Our results show that the class of standards that Maskin had identified as being implementable can be implemented (by mechanisms with no regret) even when information is asymmetric. (iv) Finally, we find that appropriate refinements of Bayesian equilibria broadens the scope mechanism design -- a fact discovered by Palfrey and Srivastava (1987b) for economies with private values and by Moore and Repullo (1988) for economies with complete information. Our results hold for more general economies including those with common values. In addition, the refinement is generated by changes made by the mechanism designer and we do not rely on agents employing a specific refinement criterion -- observe that the Palfrey and Srivastava (1987b) mechanism is based on elimination of dominated strategies and not successive elimination of dominated strategies. The latter criterion would be more appealing from a game-theoretic standpoint.

The following section presents the basic model that we shall use -- an exchange economy with privately informed agents. Section 3 introduces Bayesian equilibria with no regret. Sections 4, 5 and 6 present results on Bayesian-implementation and NR-implementation respectively and their welfare inplications. The final section discusses possible extensions.

2. Preliminaries

An *asymmetric information economy*, e, is a triple $\langle L, N, \Theta\rangle$. L is a *set of goods*, N is a *set of agents* and Θ is a *set of states of the world*. All of these sets are non-empty and finite and the cardinalities of L and N are given by ℓ and n, respectively. $\mathcal{E}$ is the domain of all asymmetric information economies. In the definitions that follow, we focus on a given $e \in \mathcal{E}$. An explicit reference to e is dropped to minimize notational burden.

Let $e = \langle L, N, \Theta\rangle$ be given. Every agent $i \in N$ is completely characterized by a list $(u_i, \omega_i, \Pi_i, q_i^*)$, where u_i: $\mathbb{R}_+^\ell \times \Theta \to \mathbb{R}$ is *agent i's von Neumann-Morgenstern utility function*; ω_i $(\neq 0) \in \mathbb{R}_+^\ell$ is *agent i's initial endowment of goods;* Π_i is *agent i's natural information partition* of Θ and q_i^*: $\Theta \to (0, 1)$ is *agent i's prior probability distribution on* Θ. Each constituent of this list is assumed to be given exogenously, and is common knowledge in the sense of Aumann (1976). Let the function I_i^o: $\Theta \to \Pi_i$ be defined by $I_i^o(\theta) \equiv \{\theta' \in \Theta$: there exists $\pi_i \in \Pi_i$ such that $\theta, \theta' \in \pi_i\}$. The latter is *agent i's natural information set in state* θ. By "natural" information we refer to the information structure that the agent is exogenously endowed with. This distinguishes it from the information that can be acquired endogenously. In the sequel, let $\Pi \equiv \times_{i\in N}\Pi_i$ and $\Omega \equiv \sum_{i\in N}\omega_i$. In addition, unless specified otherwise, $x \equiv (x_i)_{i\in N}$ and $x_{-i} \equiv (x_j)_{j\in N\setminus\{i\}}$.

$A \equiv \{z \in \mathbb{R}_+^{\ell n}$: $\sum_{i\in N}z_i \leq \Omega\}$ is the *set of feasible allocations*. A *state-contingent allocation* is a random variable f: $\Theta \to A$. F is the domain of such functions. A *performance standard* φ is a non-empty subset of F such that for all $\theta \in \Theta$ and all $f \in \varphi$, $f(\theta) \neq 0$.. Φ is the class of all performance standards.

Let $\mathbb{P}(X)$ denote the set of non-empty subsets of X. *Agent i's posterior probability distribution* is the function q_i: $\Theta \times \mathbb{P}(\Theta) \to [0, 1]$ defined by Bayes' Law, i.e. for all $\theta \in \Theta$ and for all $\mathcal{J} \in \mathbb{P}(\Theta)$,

$$q_i(\theta, \mathcal{J}) = \begin{cases} \dfrac{q_i^*(\theta)}{\Sigma_{\theta' \in \mathcal{J}} q_i^*(\theta')}, & \text{if } \theta \in \mathcal{J}; \\ 0, & \text{otherwise.} \end{cases}$$

Agent i's expected utility from $f \in F$, given $\mathcal{J} \in \mathbb{P}(I_i(\theta))$ is $\sum_{\theta' \in \mathcal{J}} q_i(\theta', \mathcal{J}) u_i(f_i(\theta'), \theta')$, and is written more compactly as $EU_i(f \mid \mathcal{J})$. *Agent i's $\mathcal{J}$-expected lower contour set at f* is given by $EL_i(f \mid \mathcal{J}) \equiv \{g \in F\text{: } EU_i(f \mid \mathcal{J}) \geq EU_i(g \mid \mathcal{J})\}$.

The domain under consideration, $\mathcal{E}$ is defined as the collection of all economies $e = \{L, N, \Theta\} \in \mathcal{E}$, that satisfy the following:

[A1] *(strict monotonicity of preferences)* $\forall i \in N$, $\forall \theta \in \Theta$, $u_i(.,.)$ is strictly increasing in $z_i \in \mathbb{R}_+^{\ell}$, and

[A2] *(non-exclusivity of information)* $\forall i \in N$, $\forall \theta \in \Theta$, $\cap_{j \in N \setminus \{i\}} I_j^o(\theta) = \{\theta\}$.

[A3] $|N| > 2$.

A *mechanism* is a game $\Gamma = \{N, M, \xi\}$, where, given that M_i is agent i's *message (or action) space*, $M \equiv \times_{i \in N} M_i$; and ξ*: $M \to A$ is an outcome function.* *Agent i's strategy* is a random variable s_i: $\Theta \to M_i$ such that s_i is Π_i-measurable. Let S_i be the domain of such functions. Let $S \equiv \times_{i \in N} S_i$.

3. Bayesian Equilibria with No Regret

The fundamental solution concept for games with asymmetric information is that of Bayesian equilibrium due to Harsanyi (1967). This concept is

defined as follows:

$s \in S$ is a *Bayesian equilibrium of* $\Gamma = \langle N, M, \xi\rangle$ if $\forall i \in N$, $\forall\theta \in \Theta$, $\forall s'_i \in S_i$,

$$\xi\circ(s'_i, s_{-i}) \in EL_i(\xi\circ s \mid I^o_i(\theta)).$$

Let $E^o(\Gamma)$ denote the *set of Bayesian equilibria of* Γ and $E^o_F(\Gamma) \equiv \{\xi\circ s \in F: s \in E^o(\Gamma), \Gamma = \langle N, M, \xi\rangle\}$.

We shall use the following structure to formally specify the changes in the rules of play introduced by a regret phase in a game. The construction given below shall be used throughout the paper. Alternative constructions are, of course, possible. The eventual objective of this paper is to provide <u>one</u> method of construction that a mechanism designer can employ.

An original game Γ played with a regret phase is denoted Γ^R.

Suppose Γ is such that $E^o(\Gamma) \neq \emptyset$. Let $K_i \equiv \{\kappa_i: E^o(\Gamma) \to M_i$ defined by: $\exists\theta \in \Theta$ such that $\forall s \in E^o(\Gamma)$, $s_i(\theta) = \kappa_i(s)\}$. Let $I_i(\theta, s) \equiv \{\theta' \in I^o_i(\theta): s_{-i}(\theta') = s_{-i}(\theta)\}$ be the *information set revealed by s in* θ.

Γ^R is played in four stages.

STAGE 1:

The agents and the mechanism designer compute $E^o(\Gamma)$.

BETWEEN STAGES 1 AND 2:

Nature selects $\theta \in \Theta$.

STAGE 2:

Each agent i submits $\kappa_i \in K_i$ to the designer. The designer makes κ public knowledge.

STAGE 3:

Each agent i is given the opportunity to entertain a desire to deviate from any $\kappa(s)$ for $s \in E^o(\Gamma)$. Let the set of Bayesian equilibria

that have not induced a desire for unilateral deviation by any $i \in N$ be denoted $E(\Gamma)$.

STAGE 4:

By some unbiased method, $s \in E(\Gamma)$ is selected and is common knowledge among the agents and the designer. The designer chooses the outcome $\xi(\kappa(s))$.

$E(\Gamma)$ is the *set of Bayesian equilibria with no regret.* Let $E_F(\Gamma) \equiv \{\xi \circ s \in F\colon s \in E(\Gamma),\ \Gamma = \langle N, M, \xi\rangle\}$. A game with a regret phase, Γ^R, for which $E(\Gamma) \neq \varnothing$ is a *mechanism with no regret.*

For the purposes of this paper, the following conditions shall suffice for determining $E(\Gamma)$. (A) is a sufficient condition for $E(\Gamma) \neq \varnothing$ and (B) is a necessary condition.

Condition (A): If $\exists s \in E^o(\Gamma)$ such that $\forall s' \in E^o(\Gamma)$, $\forall i \in N$, $\forall s''_i \in S_i$,

$$\xi \circ (s''_i, s_{-i}) \in EL_i(\xi \circ s \mid I_i(\theta, s')),$$

then $s \in E(\Gamma)$.

Condition (B): If $E(\Gamma) \neq \varnothing$, then $\forall s, s' \in E(\Gamma)$, $\forall i \in N$, $\forall s''_i \in S_i$,

$$\xi \circ (s''_i, s_{-i}) \in EL_i(\xi \circ s \mid I_i(\theta, s')).$$

Condition (A) provides a method for determining at least one Bayesian equilibrium with no regret. If there exists an equilibrium in $E^o(\Gamma)$ that is immune to information revealed by all the Bayesian equilibria, it must be a Bayesian equilibrium with no regret. Condition (B) provides an internal consistency condition for the set $E(\Gamma)$. Every Bayesian equilibrium with no regret must be immune to information revealed by any other Bayesian equilibrium with no regret. This follows from the fact that in stage four a Bayesian equilibrium with no regret will be chosen to determine the final outcome. Given that in stage two the agents do not

know which one will be chosen, they will not have an incentive to unilaterally make arbitrary submissions for any equilibrium in $E(\Gamma)$. This rules out unilateral manipulation on the part of the agents by the submission of messages that could mislead other agents. An equilibrium that is not in $E(\Gamma)$ does not convey information since agents may submit messages corresponding to such an equilibrium to mislead others about the state of the world knowing that such equilibria shall never be used to determine the final outcome.

4. Bayesian-Implementation and Welfare Implications

The classical approach to welfare economics has been to identify the subset of allocations that are Pareto-efficient within the set of all physically and technologically feasible allocations. Among these efficient allocations, attention is generally focused on ones that are individually rational. However, in asymmetric information economies, these welfare evaluations must also take account of informational constraints -- an uninformed social planner cannot identify the set of individually rational-efficient allocations in the absence of complete information about the agents' preferences. The notion of an individually rational-efficient allocation would vary depending on the extent of insurance that the allocation provides each agent. Individual rationality and Pareto-efficiency are thus extended to take account of different levels of insurance and the appropriate notion depends on the timing of the welfare analysis (see Holmström and Myerson (1983) for a detailed discussion). The two primary concepts of efficiency are:

Interim-efficiency: A state-contingent allocation f is *interim-efficient* if there is no $g \in F$ such that $\forall i \in N$, $\forall \theta \in \Theta$, $f \in EL_i(g \mid I_i^o(\theta))$ and $f \in \mathrm{int}(EL_i(g \mid I_i^o(\theta))$ for some $i \in N$ and some $\theta \in \Theta$.

Ex post-efficiency: A state-contingent allocation f is *ex post-efficient* if there is no $g \in F$ such that $\forall i \in N$, $\forall \theta \in \Theta$, $f \in EL_i(g \mid \{\theta\})$ and $f \in \mathrm{int}(EL_i(g \mid \{\theta\})$ for some $i \in N$ and some $\theta \in \Theta$.

Given $w \in F$ defined by $w(\theta) = \omega$ for all $\theta \in \Theta$, let $\mathcal{P}^i \equiv \{f \in F$: f is interim efficient and $\forall i \in N$, $\forall \theta \in \Theta$, $w \in EL_i(f \mid I_i^o(\theta))\}$ and $\mathcal{P}^e \equiv \{f \in F$: f is ex post efficient and $\forall i \in N$, $\forall \theta \in \Theta$, $w \in EL_i(f \mid \{\theta\}\}$ denote, respectively, the *sets of interim individually rational-efficient* and *ex post individually rational-efficient performance standards.*

Once a social planner decides on the appropriate notion of efficiency, the question of implementing an efficient performance standard arises. Once again, the informational asymmetry poses a constraint. A naive mechanism in which each agent is asked to report his/her private information to the planner will generally not ensure truth-telling as the unique equilibrium strategy profile. Thus, we need to be guaranteed the existence of a mechanism that implements the given standard. This is defined (for the case where Bayesian equilibrium is the solution concept) as follows:

A performance standard φ is *Bayesian-implementable by* Γ *in* $e = \{L, N, \Theta\}$ if $\emptyset \neq E_F^o(\Gamma) \subseteq \varphi$.

A performance standard φ is *Bayesian implementable* (in a global sense) if $\forall e \in \mathcal{E}$, $\exists\Gamma$ such that φ is Bayesian-implementable by Γ in e.

Remark: *Full* Bayesian-implementation of φ by Γ requires that $E_F^o(\Gamma) = \varphi$, whereas *weak* Bayesian-implementation of φ by Γ requires that $\varphi \cap E_F^o(\Gamma) \neq \emptyset$.

Next, we present a condition that is necessary and, in economies in $\mathcal{E}$,

sufficient for a performance standard to be Bayesian-implementable. Before we present the condition, some additional definitions are needed. We shall use the approach of Postlewaite and Schmeidler (1986) and Palfrey and Srivastava (1987) to define a "collection of compatible manipulation operators".

A *collection of compatible manipulation operators for* Π *(CCMO)*, denoted $\alpha = (\alpha_i)_{i \in N}$, is defined by

$$\text{(i) } \forall i \in N,\ \alpha_i: \Pi_i \to \Pi_i,$$

$$\text{(ii) } \forall \pi \in \Pi,\ \{\textstyle\bigcap_{i \in N} \pi_i \neq \emptyset\} \Rightarrow \{\textstyle\bigcap_{i \in N} \alpha_i(\pi_i) \neq \emptyset\}.$$

Let $\theta^{\alpha}: \Theta \to \Theta$ be the *deception induced by* α and defined by $\theta^{\alpha}(\theta) \equiv \bigcap_{i \in N} \alpha_i(I_i^{\circ}(\theta))$. By assumption A2, θ^{α} is a well-defined function.

A performance standard φ satisfies *Property M* if the following is true:

$\exists \varphi' \in \Phi$ such that $\varphi' \subseteq \varphi$ and $\forall f \in F$, $\forall$CCMO's α,

<u>if</u> (i) $f \in \varphi'$ and

(ii) $\forall i \in N$, $\forall\ g \in F$, $\forall \theta \in \Theta$, $\forall \theta' = \theta^{\alpha}(\theta)$, $\{g \in EL_i(f \mid I_i^{\circ}(\theta'))\} \Rightarrow \{g \circ \theta^{\alpha} \in EL_i(f \circ \theta^{\alpha} \mid I_i^{\circ}(\theta))\}$,

<u>then</u> $f \circ \theta^{\alpha} \in \varphi'$.

Remark: φ satisfies *Bayesian monotonicity* if the Property M is modified as follows: $\forall f \in F$, $\forall$CCMO's α, if (i)' and (ii)' imply $f \circ \theta^{\alpha} \in \varphi$, where (i)' $f \in \varphi$ and (ii)' is the same as (ii) in the definition above.

Fact 1: *There exists* $\varphi \in \Phi$ *such that* φ *satisfies Property M and violates Bayesian monotonicity.*

To check that this true, simply choose φ such that it is the union of a Bayesian monotonic set $\varphi' \in \Phi$ and a set $\varphi'' \in \Phi$ which is not Bayesian monotonic. For examples of such sets, see Palfrey and Srivastava (1987a).

Fact 2: *Let* $e = \{L, N, \Theta\}$ *be an economy in* $\mathcal{E}$. *A performance standard* φ *is*

fully Bayesian-implementable by a game in e if and only if it satisfies Bayesian monotonicity.

Proof: See Palfrey and Srivastava (1985). ■

The following proposition provides a parallel characterization of Bayesian-implementability.

Proposition 1: *Let $e = \{L, N, \Theta\}$ be an economy in $\mathcal{E}$. A performance standard φ is Bayesian-implementable by a game in e if and only if it satisfies Property M.*

Proof: φ is Bayesian-implementable if and only if there exists $\varphi' \in \Phi$ such that $\varphi' \subseteq \varphi$ and φ' is fully Bayesian-implementable. Given Fact 2, the proposition follows from the definitions. ■

In conjunction with the facts given above, the proposition has an interesting implication: a performance standard may be Bayesian-implementable even though it violates Bayesian monotonicity.

Palfrey and Srivastava's (1987a) examples suggest that the $\mathcal{P}^1$ and $\mathcal{P}^e$ sets are not fully Bayesian-implementable. However, disappointing as this may be, we are interested in a more crucial question: can we ever hope to design a mechanism so that all of its Bayesian equilibrium outcomes are individually rational-efficient (in either an interim or an ex post sense)? In other words, is it possible to Bayesian-implement (i.e. to fully Bayesian-implement some subset of) $\mathcal{P}^1$ or $\mathcal{P}^e$ in the domain of economies that Palfrey and Srivastava have examined? It must be noted that their result does not answer this latter question. The following results confirm that the answer to the question is, indeed, negative.

The theorems are proved by way of counterexamples. We shall present simple ones using linear economies. The negative results do not depend on

the assumption of linearity and hold for other utility specifications as well.

Theorem 1: *If $\varphi \subseteq \mathcal{P}^1$, then φ is not Bayesian-implementable.*

Proof: Consider the following example:

Example 2: We define $e = \langle L, N, \Theta\rangle$ as follows. Let $L = \{X, Y\}$ with the quantities of the two goods being denoted by the corresponding lower case letters. Let $N = \{1, 2, 3, 4\}$ and let $\Theta = \{a, b, c\}$. $\Pi_1 = \Pi_2 = \{(a), (b), (c)\}$ and $\Pi_3 = \Pi_4 = \{(a), (b, c)\}$, i.e. agents 1 and 2 are fully informed and agents 3 and 4 cannot distinguish between states b and c when one of them occurs. An allocation z is written as $(x_i, y_i)_{i\in N}$. $\omega = ((0, 1), (0, 1), (1, 0), (1, 0))$. Each state is equally likely, i.e. $q_i^*(a) = q_i^*(b) = q_i^*(c) = \frac{1}{3}$ for all $i \in N$. $u_i: \mathbb{R}_+^{\ell} \times \Theta \to \mathbb{R}$ is given as follows:

$$\forall i \in \{1, 2\}, \forall\theta \in \Theta, \qquad u_i(z_i, \theta) = \qquad x_i + 1.1y_i,$$

$$u_3(z_3, \theta) = \begin{cases} (x_3 + y_3), & \text{if } \theta = a \\ 0.25(x_3 + y_3) & \text{if } \theta = b \\ 0.75(x_3 + y_3) & \text{if } \theta = c. \end{cases}$$

$$u_4(z_4, \theta) = \begin{cases} (x_4 + y_4), & \text{if } \theta = a \\ 0.75(x_4 + y_4) & \text{if } \theta = b \\ 0.25(x_4 + y_4) & \text{if } \theta = c. \end{cases}$$

It can be checked that $\mathcal{P}^1 \neq \emptyset$. Consider the following state-contingent allocation, denoted f^*:

	a	b	c
$z_1 =$	(0, 1)	(0, 1)	(0, 1)
$z_2 =$	(0, 1)	(0, 1)	(0, 1)
$z_3 =$	(1, 0)	(0, 0)	$(\frac{5}{3}, 0)$
$z_4 =$	(1, 0)	(2, 0)	$(\frac{1}{3}, 0)$

It can be checked that $f^* \in \mathcal{P}^1$. Also check that assumptions A1–A2 are satisfied.

Choose any $\varphi \in \Phi$ such that $\varphi \subseteq \mathcal{P}^1$. We shall show that the hypothesis of Property M is satisfied. Consider the function $\alpha_i: \Pi_i \rightarrow \Pi_i$ for all $i \in N$ defined by $\alpha_i(\pi_i) = \{a\}$ for all $i \in N$ and all $\pi_i \in \Pi_i$. Thus, α is a CCMO and for all $\theta \in \Theta$, $\theta^{\alpha}(\theta) = a$. Pick $f \in \varphi$ and $g \in F$. Write $f(a)$ as (x, y) and $g(a)$ as (x', y'). By construction, the utility functions of agents 1 and 2 are state-independent. Therefore, given that they are completely informed, the hypothesis of Property M is trivially satisfied for these two agents. The following relationships imply that the hypothesis of Property M is met for the remaining agents:

$x_3 + y_3 \geq x_3' + y_3' \Rightarrow 0.5[0.25(x_3 + y_3) + 0.75(x_3 + y_3)] \geq 0.5[0.25(x_3' + y_3') + 0.75(x_3' + y_3')]$.

$x_4 + y_4 \geq x_4' + y_4' \Rightarrow 0.5[0.75(x_4 + y_4) + 0.25(x_4 + y_4)] \geq 0.5[0.75(x_4' + y_4') + 0.25(x_4' + y_4')]$.

For $\mathcal{P}^1$ to satisfy Property M, we must have $f \circ \theta^{\alpha} \in \varphi$. $f \circ \theta^{\alpha}$ recommends the following allocation in states b and c for agents 3 and 4:

	b	c
Agent 3	(x_3, y_3)	(x_3, y_3)
Agent 4	(x_4, y_4)	(x_4, y_4)

By interim individual rationality of f, $(x_3, y_3) \neq (0, 0)$. By the construction of this example and given that $f \in \varphi \subseteq \mathcal{P}^1$, $x_3 > 0$ and $x_4 > 0$.

Choose ε such that $\min\{x_3, x_4\} > \varepsilon > 0$. Consider an alternative rule $h \in F$ defined by

$$h(a) = f(a),$$

$$\forall i \in \{1, 2\}, \forall \theta \in \{b, c\}, h_i(\theta) = f_i(a),$$

$$h_3(b) = (x_3 - \varepsilon, y_3), \qquad h_3(c) = (x_3 + \tfrac{\varepsilon}{3}, y_3),$$

$$h_4(b) = (x_4 + \varepsilon, y_4), \qquad h_4(c) = (x_4 - \tfrac{\varepsilon}{3}, y_4).$$

Thus, $EU_3(h \mid \{b, c\}) = 0.5[0.25(x_3 - \varepsilon + y_3) + 0.75(x_3 + \frac{\varepsilon}{3} + y_3)] = 0.5[0.25(x_3 + y_3) + 0.75(x_3 + y_3)] = EU_3(f \circ \theta^{\alpha} \mid \{b, c\})$. Trivially, for $i \in \{1, 2\}$ and all $\theta \in \Theta$, $EU_i(h \mid \{\theta\}) = EU_i(f \circ \theta^{\alpha} \mid \{\theta\})$. Also, trivially, for $i \in \{3, 4\}$, $EU_i(h \mid \{a\}) = EU_i(f \circ \theta^{\alpha} \mid \{a\})$. However, $EU_4(h \mid \{b, c\}) = 0.5[0.75(x_4 + \varepsilon + y_4) + 0.25(x_4 - \frac{\varepsilon}{3} + y_4)] > 0.5[0.25(x_4 + y_4) + 0.75(x_4 + y_4)] = EU_4(f \circ \theta^{\alpha} \mid \{b, c\})$. Thus, $f \circ \theta^{\alpha} \notin \mathcal{P}^{i}$.

For any $\varphi \subseteq \mathcal{P}^{i}$ with $f \in \varphi$, we find that $f \circ \theta^{\alpha} \notin \varphi$. Thus, $\mathcal{P}^{i}$ violates Property M. By Proposition 1, it is not Bayesian-implementable in e. ∎

Theorem 2: *If $\varphi \subseteq \mathcal{P}^{e}$, then φ is not Bayesian-implementable.*

<u>Proof:</u> Consider the following example.

Example 3: Consider an economy e' which retains all the specifications of e from Example 2 above with one exception: agent 3's utility function is altered as follows:

$$u_3(z_3, \theta) = \begin{cases} (x_3 + y_3), & \text{if } \theta = a \\ (x_3 + 1.5y_3) & \text{if } \theta = b \\ (x_3 + 0.5y_3) & \text{if } \theta = c. \end{cases}$$

Given these specifications, $\mathcal{P}^{e} \neq \emptyset$. Choose $\alpha_i: \Pi_i \to \Pi_i$ as in the previous proof, i.e. for all $i \in N$, for all $\pi_i \in \Pi_i$, $\alpha_i(\pi_i) = \{a\}$. Thus, α is a CCMO and for all $\theta \in \Theta$, $\theta^{\alpha}(\theta) = a$. Choose any $\varphi \in \Phi$ such that $\varphi \subseteq \mathcal{P}^{e}$ and

pick $f \in \varphi$. Next, we establish that the hypothesis of Property M is met. From the relationships derived in the proof of Theorem 1, we know that the relevant relationships for agents 1, 2 and 4 hold. To check that the relevant relationship holds for agent 3, consider the following observations:

Pick $f \in \varphi$ and $g \in F$. Write $f(a)$ as (x, y), and $g(a)$ as (x', y'). Thus, we have

$x_3 + y_3 \geq x'_3 + y'_3 \Rightarrow 0.5[(x_3 + 1.5y_3) + (x_3 + 0.5y_3)] \geq 0.5[(x'_3 + 1.5y'_3) + (x'_3 + 0.5y'_3)]$.

Thus, the hypothesis of Property M is satisfied. For $\mathcal{P}^e$ to satisfy Property M, we must have $f \circ \theta^\alpha \in \varphi$. $f \circ \theta^\alpha$ recommends the following allocation in states a and b for agents 1 and 3:

	a	b
Agent 1	(x_1, y_1)	(x_1, y_1)
Agent 3	(x_3, y_3)	(x_3, y_3)

By ex post individual rationality of f, $(x_3, y_3) \neq (0, 0)$. By the construction of this example and given that $f \in \varphi \subseteq \mathcal{P}^e$, $x_3 > 0$ and $y_1 > 0$. Choose ε such that $\min\{\frac{10}{11}x_3, y_1\} > \varepsilon > 0$. Consider an alternative rule $h \in F$ defined by:

$h_1(b) = (x_1+1.1\varepsilon, y_1-\varepsilon)$

$h_3(b) = (x_3-1.1\varepsilon, y_3+\varepsilon)$

$\forall i \in \{1, 3\}, \forall \theta \in \{a, c\}, h_i(\theta) = f_i(a)$

$\forall i \in \{2, 4\}, h_i = f_i \circ \theta^\alpha$.

Thus, $EU_3(h \mid \{b\}) = x_3 - 1.1\varepsilon + 1.5\varepsilon + 1.5y_3 = x_3 + 0.4\varepsilon + 1.5y_3 > x_3 + 1.5y_3 = EU_3(f \circ \theta^\alpha \mid \{b\})$ and $EU_1(h \mid \{b\}) = x_1 + 1.1\varepsilon + 1.1y_1 - 1.1\varepsilon = x_1 + 1.1y_1 = EU_1(f \circ \theta^\alpha \mid \{b\})$. For all $i \in N$ and all $\theta \in \Theta$, $f \circ \theta^\alpha \in EL_i(h \mid \{\theta\})$. Thus, $f \circ \theta^\alpha \notin \varphi \subseteq \mathcal{P}^e$. Given that this holds for all $\varphi \subseteq \mathcal{P}^e$, $\mathcal{P}^e$ does not

satisfy Property M. By Proposition 1, it is not Bayesian-implementable in e'. ■

5. No Regret Implementation

In this section, we discuss the implementability properties of mechanisms with no regret and provide necessary and sufficient conditions for this method of implementation.

A performance standard φ is *No regret-implementable (NR-implementable) by* Γ^R *in* $e = \{L, N, \Theta\}$ if $\emptyset \neq E_F(\Gamma) \subseteq \varphi$.

A performance standard φ is *NR-implementable* (in a global sense) if $\forall e \in \mathcal{E}$, $\exists \Gamma$ such that φ is NR-implementable by Γ^R in e.

Remark: φ is *fully NR-implementable by* Γ^R if $E_F(\Gamma) = \varphi$.

Next, we define some properties that shall be used to characterize NR-implementable standards.

A performance standard φ satisfies *Property M1 with respect to* $\Gamma = \{N, M, \xi\}$ if the following is true:

$\exists \varphi' \in \Phi$ such that $\varphi' \subseteq \varphi$ and $\forall f \in F$, $\forall$CCMO's α,

<u>if</u> (i) $f \in \varphi'$

(ii) $\forall i \in N$, $\forall g \in F$, $\forall \theta \in \Theta$, $\forall \theta' = \theta^{\alpha}(\theta)$, $\{\forall s \in E(\Gamma),\ g \in EL_i(f \mid I_i^{o}(\theta')) \bigcap EL_i(f \mid I_i(\theta', s))\} \Rightarrow \{\forall s' \in S,\ g \circ \theta^{\alpha} \in EL_i(f \circ \theta^{\alpha} \mid I_i^{o}(\theta)) \bigcap EL_i(f \circ \theta^{\alpha} \mid I_i(\theta, s'))\}$,

<u>then</u> $f \circ \theta^{\alpha} \in \varphi'$.

A performance standard φ satisfies *Property M2 with respect to* Γ if $\forall f \in F$, $\forall$CCMO's α, (i)' and (ii)' imply $f \circ \theta^{\alpha} \in \varphi$, where (i)' $f \in \varphi$ and (ii)' is the same as (ii) in the definition above.

Theorem 3: *Let $e = \{L, N, \Theta\}$ be an economy in $\mathcal{E}$, and let φ be a performance standard. If φ is NR-implementable by Γ^R in e, then φ satisfies Property M1 with respect to Γ.*

<u>Proof:</u> By definition of NR-implementation, for some game $\Gamma = \langle N, M, \xi \rangle$, there exists $\varphi' \subseteq \varphi$ such that φ' is fully NR-implemented by Γ^R. Thus, for all $f \in \varphi'$ there exists $s \in E(\Gamma)$ that $f = \xi \circ s$. By Condition (B) the following holds for all $i \in N$, for all $\theta \in \Theta$, for all $s'_i \in S_i$, for all $s'' \in E(\Gamma)$:

$$\xi \circ (s'_i, s_{-i}) \in EL_i(f \mid I^o_i(\theta)) \bigcap EL_i(f \mid I_i(\theta, s'')) \tag{1}$$

Choose a CCMO α. Next, suppose that for all $i \in N$, for all $\theta \in \Theta$, for all $\theta' = \theta^{\alpha}(\theta)$, for all $s'' \in E(\Gamma)$, for all $g \in EL_i(f \mid I^o_i(\theta')) \bigcap EL_i(f \mid I_i(\theta', s''))$, the following holds for all $s^* \in S$:

$$g \circ \theta^{\alpha} \in EL_i(f \circ \theta^{\alpha} \mid I^o_i(\theta)) \bigcap EL_i(f \circ \theta^{\alpha} \mid I_i(\theta, s^*)) \tag{2}$$

Given (1) and (2), for all $i \in N$, for all $\theta \in \Theta$, for all $s'_i \in S_i$, for all $s^* \in S$, the following holds:

$$\xi \circ (s'_i, s_{-i}) \circ \theta^{\alpha} \in EL_i(f \circ \theta^{\alpha} \mid I^o_i(\theta)) \bigcap EL_i(f \circ \theta^{\alpha} \mid I_i(\theta, s^*)) \tag{3}$$

By definition of α, $s_i \circ \theta^{\alpha}$ is Π_i-measurable for all $i \in N$. By Condition (A), (3) implies that $s \circ \theta^{\alpha} \in E(\Gamma)$ and $f \circ \theta^{\alpha} \in E_F(\Gamma)$. By definition of full NR-implementation of φ', $E_F(\Gamma) \subseteq \varphi'$. Thus, $f \circ \theta^{\alpha} \in \varphi'$ and φ satisfies Property M1. ■

Theorem 4: *Let $e = \{L, N, \Theta\}$ be an economy in $\mathcal{E}$ and let φ be a performance standard. There exists Γ such that if φ satisfies Property M2 with respect to Γ, then φ is NR-implementable by Γ^R in e.*

<u>Proof:</u> The proof of this statement derives from the following lemmata.

Lemma 1: *There exist Γ and $s \in E(\Gamma)$ such that for all $i \in N$, for all $\theta \in \Theta$,*

$I_1(\theta, s) = \{\theta\}$.

Lemma 2: *There exists* Γ *such that (i)* Γ *satisfies Lemma 1 and (ii) if* φ *satisfies Property M2 with respect to* Γ*, then* $E_F(\Gamma) \subseteq \varphi$.

The proofs of these results are given in the appendix. ■

Corollary to Theorem 4: *Let* $e = \{L, N, \Theta\}$ *be an economy in* $\mathcal{E}$*, and let* φ *be a performance standard. If* φ *is fully NR-implementable by* Γ^R *in e, then* φ *satisfies Property M2 with respect to* Γ*. Conversely, there exists* Γ *such that if* φ *satisfies Property M2 with respect to* Γ*, then* φ *is fully NR-implementable by* Γ^R *in e.*

6. No Regret Implementation and Welfare Implications

This section explores whether or not the individual rationality-efficiency performance standards are implementable by no regret mechanisms. We have bad news and good news. The bad news is that the interim standard is not NR-implementable. The good news is that the ex post standard is. Often we are not interested in implementing the entire ex post individual rationality-efficiency set. Instead, we would like to find out which subsets of this set are implementable. This brings us to the most significant result: *any* performance standard that is Nash-implementable is NR-implementable. Hence, extremely important economic performance standards such as the *core* or the *Walrasian correspondence* are implementable by mechanisms with no regret even in asymmetric information economies.

Theorem 5: $\mathcal{P}^1$ *is not NR-implementable.*

Proof: Suppose that the theorem were not true, i.e. for all $e \in \mathcal{E}$, there exists a game Γ such that $\mathcal{P}^1$ is NR-implementable by Γ^R in e. Consider the economy e defined in Example 2 (see proof of Theorem 1 above). By definition of NR-implementation, for some $\Gamma = \{N, M, \xi\}$ in e, there exists $\varphi \subseteq \mathcal{P}^1$ such that for all $f \in \varphi$, there exists $s \in E(\Gamma)$ with $\xi \circ s = f$. In addition, check that in the economy e in Example 2, the hypotheses of Property M1 are satisfied with respect to Γ. Choose a CCMO α as in Example 2. Pick $f \in \varphi$ and $g \in F$. Write $f(a)$ as (x, y) and $g(a)$ as (x', y'). By construction, the utility functions of agents 1 and 2 are state-independent. Therefore, given that they are completely informed, the hypothesis of Property M1 is trivially satisfied for these two agents. The following relationships imply that the hypothesis of Property M1 is met for the remaining agents:

$$x_3 + y_3 \geq x_3' + y_3' \Rightarrow 0.5[0.25(x_3 + y_3) + 0.75(x_3 + y_3)] \geq 0.5[0.25(x_3' + y_3') + 0.75(x_3' + y_3')].$$

$$x_4 + y_4 \geq x_4' + y_4' \Rightarrow 0.5[0.75(x_4 + y_4) + 0.25(x_4 + y_4)] \geq 0.5[0.75(x_4' + y_4') + 0.25(x_4' + y_4')] \quad \text{and}$$

$$x_3 + y_3 \geq x_3' + y_3' \Rightarrow 0.25(x_3 + y_3) \geq 0.25(x_3' + y_3')$$

$$x_3 + y_3 \geq x_3' + y_3' \Rightarrow 0.75(x_3 + y_3) \geq 0.75(x_3' + y_3')$$

$$x_4 + y_4 \geq x_4' + y_4' \Rightarrow 0.75(x_4 + y_4) \geq 0.75(x_4' + y_4')$$

$$x_4 + y_4 \geq x_4' + y_4' \Rightarrow 0.25(x_4 + y_4) \geq 0.25(x_4' + y_4')$$

Given these observations, the hypothesis of Property M1 is met for any game Γ. The rest of the proof of Theorem 5 follows the proof of Theorem 1 and it is established that $f \circ \theta^\alpha \notin \varphi$. Thus, $\mathcal{P}^1$ does not satisfy Property M1 with respect to Γ. Given Theorem 3 and the assumption that $\mathcal{P}^i$ is NR-implementable by Γ^R, we have a contradiction. ■

Theorem 6: *$\mathcal{P}^e$ is NR-implementable.*

This result follows from Lemma 3 and a more general result given in Theorem 7. The lemma is proved in the appendix. We shall use the following definition:

A performance standard φ satisfies *Property M^** if the following is true:

$\exists\varphi' \in \Phi$ such that $\varphi' \subseteq \varphi$ and $\forall f \in F$, $\forall$CCMO's α,

<u>if</u> (i) $f \in \varphi'$ and

(ii) $\forall i \in N$, $\forall\ g \in F$, $\forall\theta \in \Theta$, $\forall\theta' = \theta^{\alpha}(\theta)$, $\{g \in EL_i(f \mid \{\theta'\})\} \Rightarrow \{g \circ \theta^{\alpha} \in EL_i(f \circ \theta^{\alpha} \mid \{\theta\})\}$,

<u>then</u> $f \circ \theta^{\alpha} \in \varphi'$.

Lemma 3: *$\mathcal{P}^e$ satisfies Property M^*.*

Theorem 7: *Let φ be a performance standard. If φ is Bayesian-implementable (Nash-implementable) in economies with complete information (i.e. for all $i \in N$ and all $\theta \in \Theta$, $I_i^O(\theta) = \{\theta\}$), then φ is NR-implementable.*

<u>Proof:</u> By Proposition 1, if φ is Bayesian-implementable in economies with complete information, it must satisfy Property M^*. By definition, if φ satisfies Property M^*, then there exists $\varphi' \subseteq \varphi$ which satisfies Property M2 with respect to any game Γ such that for all $i \in N$ and all $\theta \in \Theta$, there exists $s \in E(\Gamma)$ with $I_i(\theta, s) = \{\theta\}$. By Lemma 1 and Lemma 2 and the Corollary to Theorem 4, φ' is fully NR-implementable. Thus, φ is NR-implementable. ■

7. Extensions

In many economic problems of interest, initial endowments are not specified (for example, when resources are owned collectively) or a social planner may wish to efficiently allocate resources in an "equitable" manner. In such situations, we may want to replace the individual rationality condition with some equity requirement, such as *freedom from envy* (Foley (1967). The results reported in this paper, both and positive ones, hold for the corresponding problem of implementing the interim and ex post envy-free-efficient performance standards.

To summarize, we have shown that it is impossible to design "interesting" efficient mechanisms under asymmetric information using Bayesian equilibrium as the solution concept. Yet under complete information such mechanisms are possible. By employing mechanisms with no regret, similar to ones introduced by Green and Laffont (1987), we have a solution to the problem. Such mechanisms provide channels for endogenous leakage of information. In general, any success that has been achieved by Nash-implementation theory for complete information economies can be mimicked in situations where information is non-exclusive and incomplete.

AUMANN, R. (1976): "Agreeing to Disagree," *Annals of Statistics,* 4, 1236-1239.

BLUME, L. AND D. EASLEY (1987): "Implementation of Expectations Equilibria," mimeo Cornell University.

DASGUPTA, P., P. HAMMNOND AND E. MASKIN (1979): "The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility," *Review of Economic Studies,* 46, 185-216.

FOLEY, D. (1967): "Resource Allocation and the Public Sector," *Yale Economic Essays,* 7, 45-98.

GREEN, J. AND J.-J. LAFFONT (1987): "Posterior Implementability in a Two-Person Decision Problem," *Econometrica,* 55, 69-94.

HARRIS, M. AND R. TOWNSEND (1981): "Resource Allocation under Asymmetric Information," *Econometrica,* 49, 231-259.

HARSANYI, J. (1967): " Games with Incomplete Information Played by 'Bayesian' Players," *Management Science,* 14, 159-189, 320-334, 486-502.

HOLMSTRÖM, B. AND R. MYERSON (1983): "Efficient and Durable Decision Rules with Incomplete Information," *Econometrica,* 51, 1799-1820.

MASKIN, E. (1977): "Nash Equilibrium and Welfare Optimality," mimeo, M.I.T..

MOORE, J. AND R. REPULLO (1988): "Subgame Perfect Implementation," *Econometrica* (forthcoming).

MYERSON, R. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica,* 47, 61-74.

PALFREY, T. AND S. SRIVASTAVA (1985): "Implementation and Incomplete Information," mimeo, Carnegie-Mellon University.

PALFREY, T. AND S. SRIVASTAVA (1987a): "On Bayesian Implementable Allocations," *Review of Economic Studies,* 54, 193-208.

PALFREY, T. AND S. SRIVASTAVA (1987b): "Mechanism Design with Incomplete Information: A Solution to the Implementation Problem," mimeo California Institute of Technology and Carnegie-Mellon University.

POSTLEWAITE, A. AND D. SCHMIEDLER (1986): "Implementation in Differential Information Economies," *Journal of Economic Theory,* 39, 14-33.

SAIJO, T. (1988): "Strategy Space Reduction in Maskin's Theorem: Sufficient Conditions for Nash-implementation," *Econometrica,* (forthcoming).

In this appendix, we shall prove the lemmata presented in the main body of the paper. For this purpose, we shall devise an algorithm which generates a game for every performance standard. Let $\mathcal{G}$ denote the algorithm and let $\mathcal{G}(\varphi)$ denote the game that is generated when a particular φ is applied to the algorithm given below. For all $\varphi \in \Phi$, $\mathcal{G}(\varphi) = \{N, M, \xi\}$ is defined as follows:

(I) $\forall i \in N$, $M_i = \{m_i = (\pi_i(i), f(i), \delta(i)) \in \Pi_i \times F \times \mathbb{R}_+\}$.

Definition: $\forall i \in N$, m_{-i} satisfies *Property* $\gamma|i$ if the following conditions hold:

(i) $\bigcap_{j \in N \setminus \{i\}} \pi_j(j) \neq \emptyset$.

(ii) $\exists f \in \varphi$ such that $\forall j \in N \setminus \{i\}$, $f(j) = f$.

(iii) $\forall j \in N \setminus \{i\}$, $\delta(j) = 0$.

Definition: $\forall m \in M$, $\theta^*(m)$ is defined such that $\bigcap_{i \in N} \pi_i(i) = \{\theta^*(m)\}$.

Definition: $\forall i \in N$, $\forall m_{-i} \in M_{-i}$, $\theta_i(m_{-i})$ is defined such that $\bigcap_{j \in N \setminus \{i\}} \pi_j(j) = \{\theta_i(m_{-i})\}$.

Definition: $\forall m \in M$, $K(m) \equiv \{i \in N: \forall j \in N \setminus \{i\}, \delta(i) \geq \delta(j)\}$.

(II) $\xi: M \to A$ is given by the schematic diagram in Figure 3.

Proof of Lemma 1: Choose $f \in \varphi$. We shall show that there exists $s \in E(\mathcal{G}(\varphi))$ such that $\xi \circ s = f$ and for all $i \in N$ and all $\theta \in \Theta$, $I_i(\theta, s) = \{\theta\}$. Construct a strategy list $s \in S$ as follows: for all $i \in N$, for all $\theta \in \Theta$, $s_i(\theta) = (I_i(\theta), f, 0)$. It may be checked that for all $i \in N$, for all $\theta \in \Theta$, $s_{-i}(\theta)$ satisfies Property $\gamma|i$. By construction, Case 1 applies and $\xi \circ s = f$.

Consider a unilateral deviation to $s'_i \in S_i$ by agent i. We write $s'_i = (s'_{i1}, s'_{i2}, s'_{i3})$. Note that by assumption A2, for all $\theta \in \Theta$, $\theta_i(s_{-i}(\theta)) = \theta$. For all $\theta \in \Theta$, there are two possibilities:

(a) $s'_{i2}(\theta) = f$, in which case either Case 1 or Case 2 applies and $\xi(s'_i(\theta), s_{-i}(\theta)) \in \{f(\theta), 0\}$.

(b) $s'_{i2}(\theta) \neq f$. Case 3 applies and $\xi(s'_i(\theta), s_{-i}(\theta)) \in \{f'(i)(\theta), 0\}$.

By strict monotonicity of preferences, $u_i(f, \theta) \geq u_i(0, \theta)$ for all $\theta \in \Theta$. By construction for all $j \in N$, $s_{j1}(\theta) = I^o_j(\theta)$. In conjunction with assumption A2, this implies that for all $\theta \in \Theta$, for all $\theta' \in I^o_i(\theta)\backslash\{\theta\}$, there exists $j \in N\backslash\{i\}$ such that $s_{j1}(\theta) \neq s_{j1}(\theta')$. Thus, for all $i \in N$, for all $\theta \in \Theta$, $I_i(\theta, s) = \{\theta\}$. If possibility (b) occurs, by Case 3, we have $\xi \circ (s'_i, s_{-i}) \in EL_i(f \mid \{\theta\})$ which implies that for all $s'' \in S$, $\xi \circ (s'_i, s_{-i}) \in EL_i(f \mid I^o_i(\theta)) \bigcap EL_i(f \mid I_i(\theta, s''))$. By Condition (A), $s \in E(\mathcal{G}(\varphi))$. Given $\xi \circ s = f \in \varphi$, we have $E(\mathcal{G}(\varphi)) \neq \varnothing$. ■

Proof of Lemma 2: By Lemma 1, $E(\mathcal{G}(\varphi)) \neq \varnothing$. We need to show that $E_F(\mathcal{G}(\varphi)) \subseteq \varphi$.

Step 1: The proof of this lemma makes use of the following result:

Lemma 2.1: *Let $s \in E(\mathcal{G}(\varphi))$. For all $\theta \in \Theta$, $s(\theta)$ satisfies Case 1.*

Proof of Lemma 2.1: We shall establish that there cannot be a state $\theta \in \Theta$

such that $s(\theta) = (\pi_i(i),\ f(i),\ \delta(i))_{i \in N}$ satisfies either of the cases 2, 3 or 4. We write s_i as (s_{i1}, s_{i2}, s_{i3}) where the components of this list denote the functions induced by restricting the range of s_i to Π_i, F and $\mathbb{R}_+$ respectively.

Suppose that for some $\theta \in \Theta$, $s(\theta)$ satisfies one of the following: Case 2, Case 3 or Case 4. A contradiction will be established. Consider a unilateral deviation by agent $i \in N$ to $s'_i \in S_i$ such that $s'_i = (s_{i1}, s_{i2}, s'_{i3})$. s'_{i3} is such that for all $j \in N\setminus\{i\}$, for all $\theta' \in \Theta$, $s_{j3}(\theta') < s'_{i3}(\theta')$. Let $m = s(\theta)$ and $m' = (s'_i(\theta),\ s_{-i}(\theta))$. We shall show that there exists $i \in N$ such that the following hold:

$$\xi_i(m') > \xi_i(m) \qquad (*)$$

$$\forall\theta' \in \Theta,\ \xi_i(s'_i(\theta'),\ s_{-i}(\theta')) \geq \xi_i(s(\theta')) \qquad (**)$$

There are two possibilities:

(i) *Possibility 1:* There exists $j \in N$ such that $K(m) = \{j\}$. Therefore, for all $i \in N\setminus\{j\}$, m_{-i} does not satisfy Property $\gamma|i$. Choose $i \in N\setminus\{j\}$. By definition, Property $\gamma|i$ is not met even if i deviates to s'_i. By the outcome rule associated with Case 4A, we have $\xi_i(m') = \Omega$. Since $|N\setminus\{j\}| > 1$, there exists $i \in N\setminus\{j\}$ such that $\xi_i(m) \neq \Omega$. Thus, (*) holds.

(ii) *Possibility 2:* There does not exist any $j \in N$ such that $K(m) = \{j\}$. In this case, by construction, $\xi(m) = 0$. By the outcome rules associated with Cases 2A, 3A and 4A, and given that for all $\theta' \in \Theta$, for all $f \in \varphi$, $f(\theta') \neq 0$, there exists $i \in N$ such that $\xi_i(m') > 0$. Thus, (*) holds.

To check that (**) is true for the agent i for whom (*) holds, choose $\theta'' \in \Theta$ with $\theta'' \neq \theta$. There are, again, two possibilities:

(a) *Possibility 1:* There exists $j \in N$ such that $K(s(\theta'')) = \{j\}$. The arguments given in (i) above apply. Thus, for all $k \in N\setminus\{j\}$, $\xi_k(s'_k(\theta''),$

$s_{-k}(\theta''))$ = Ω. Also, $\xi(s'_j(\theta''), s_{-j}(\theta'')) = \xi(s(\theta''))$. Thus, (**) holds for i.

(b) *Possibility 2:* There does not exist $j \in N$ such that $K(s(\theta'')) = \{j\}$. By construction, for some $f \in \varphi$, $\xi(s(\theta'')) \in \{f, 0\}$. By the outcome rules associated with Cases 2A, 3A and 4A, we conclude that $\xi(s'_i(\theta''), s_{-i}(\theta'')) \in \{f, \Omega\}$. Thus, (**) holds for i.

Given that (*) and (**) are true, and given strict monotonicity of preferences, we conclude that if $s(\theta)$ does not satisfy Case 1, for some $\theta \in \Theta$, then $s \notin E(\mathcal{G}(\varphi))$. This contradicts the assumption that $s \in E(\mathcal{G}(\varphi))$. ∎

<u>Step 2:</u> Choose $s \in E(\mathcal{G}(\varphi))$. By Lemma 2.1, for all $\theta \in \Theta$, $s(\theta)$ must satisfy Case 1. We write s_i as (s_{i1}, s_{i2}, s_{i3}). Thus, for all $\theta \in \Theta$, $\bigcap_{i\in N} s_{i1}(\theta) \neq \emptyset$ and $\theta^*(s(\theta))$ is well-defined. By Π_i-measurability of s_i, $(s_{i1})_{i\in N}$ defines a CCMO and we shall write it as α, where for all $i \in N$, for all $\theta \in \Theta$, $\alpha_i(I_i(\theta)) \equiv s_{i1}(\theta)$. Observe that for all $\theta \in \Theta$, $\theta^*(s(\theta)) = \theta^\alpha(\theta)$. By construction, for all $\theta \in \Theta$, there exists $f^* \in \varphi$ such that $s_{i2}(\theta) = f^*$. This implies that there exists $f \in \varphi$ such that for all $\theta \in \Theta$, $\xi(s(\theta)) = f(\theta^*(s(\theta)))$, i.e. $\xi\circ s = f\circ\theta^\alpha$. We need to show that $f\circ\theta^\alpha \in \varphi$.

We shall first show that for all $i \in N$, for all $\theta \in \Theta$, for all $\theta' = \theta^\alpha(\theta)$, if $g \in EL_i(f \mid I_i(\theta', s^*))$ for all $s^* \in E(\mathcal{G}(\varphi))$, then $g\circ\theta^\alpha \in EL_i(f\circ\theta^\alpha \mid I_i^\circ(\theta)) \bigcap EL_i(f\circ\theta^\alpha \mid I_i(\theta, \hat{s}))$ for all $\hat{s} \in S$. In the case where $f = g$, this is trivially true. To show that this is true even when $f \neq g$, choose $i \in N$ and $g \in F$ such that $g \neq f$ and for all $\theta \in \Theta$ and for all $\theta' = \theta^\alpha(\theta)$, for all $s^* \in E(\mathcal{G}(\varphi))$, $g \in EL_i(f \mid I_i(\theta', s^*))$. By Lemma 1, there exists $s^* \in E(\mathcal{G}(\varphi))$ such that for all $\theta \in \Theta$, $I_i(\theta, s^*) = \{\theta\}$. Thus, $g \in EL_i(f \mid I_i(\theta', s^*))$ for all $s^* \in E(\mathcal{G}(\varphi))$ implies that $g \in EL_i(f \mid \{\theta'\})$. Suppose i unilaterally deviates to $s'_i \in S_i$ where for all $\theta \in \Theta$, $s'_i(\theta)$ =

$(s_{i1}(\theta),\ g,\ \delta'(i))$. $\delta'(i)$ is such that for all $j \in N\backslash\{i\}$, for all $\theta \in \Theta$, $\delta'(i) > s_{j3}(\theta)$. By construction, for all $\theta \in \Theta$, $\theta_i(s_{-i}(\theta)) = \theta^*(s(\theta))$. Thus, for all $\theta \in \Theta$, $(s'_i(\theta),\ s_{-i}(\theta))$ satisfies Case 3A and $\xi((s'_i(\theta),\ s_{-i}(\theta)) = g(\theta_i(s_{-i}(\theta))) = g(\theta^*(s(\theta)) = g(\theta^\alpha(\theta))$. Observe that, by Condition (B), $s \in E(\mathcal{G}(\varphi))$ implies that for all $\theta \in \Theta$, for all $s^* \in E(\mathcal{G}(\varphi))$, $g\circ\theta^\alpha \in EL_i(f\circ\theta^\alpha \mid I_i^\circ(\theta)) \bigcap EL_i(f\circ\theta^\alpha \mid I_i(\theta,\ s^*))$. By Lemma 1, there exists $s^* \in E(\mathcal{G}(\varphi))$ such that $I_i(\theta,\ s^*) = \{\theta\}$ for all $\theta \in \Theta$. Thus, $g\circ\theta^\alpha \in EL_i(f\circ\theta^\alpha \mid \{\theta\})$ for all $\theta \in \Theta$ and we conclude that for all $\hat{s} \in S$, $g\circ\theta^\alpha \in EL_i(f\circ\theta^\alpha \mid I_i^\circ(\theta)) \bigcap EL_i(f\circ\theta^\alpha \mid I_i(\theta,\ \hat{s}))$.

Given that the last conclusion holds for all $i \in N$, by the fact that φ satisfies Property M2 with respect to $\mathcal{G}(\varphi)$, we have $f\circ\theta^\alpha \in \varphi$. ■

<u>Proof of Lemma 3</u>: Choose $f \in \mathcal{P}^e$ and a CCMO α. Suppose that for all $i \in N$, for all $\theta \in \Theta$, for all $\theta' = \theta^\alpha(\theta)$, if $f' \in EL_i(f \mid \{\theta'\})$, then $f'\circ\theta^\alpha \in EL_i(f\circ\theta^\alpha \mid \{\theta\})$.

Next, suppose that $f\circ\theta^\alpha \notin \mathcal{P}^e$. We shall prove that this yields a contradiction. $f\circ\theta^\alpha \notin \mathcal{P}^e$ implies either one or both of the following possibilities:

Possibility A: $f\circ\theta^\alpha$ is not individually rational, i.e. given $w \in F$ defined by $w(\theta) = \omega$ for all $\theta \in \Theta$, there exists $j \in N$ and $\hat{\theta} \in \Theta$ such that $w \notin EL_j(f\circ\theta^\alpha \mid \{\hat{\theta}\})$.

Possibility B: $f\circ\theta^\alpha$ is not ex post efficient, i.e. there exists $g,\ h \in F$ such that $h \equiv g\circ\theta^\alpha$ with $h \notin EL_k(f\circ\theta^\alpha \mid \{\theta^*\})$ for some $k \in N$ and some $\theta^* \in \Theta$ and $f\circ\theta^\alpha \in EL_i(h \mid \{\theta'\})$ for all $i \in N$ and all $\theta' \in \Theta$.

By assumption, for all $i \in N$, for all $\theta \in \Theta$, for all $\theta'' = \theta^\alpha(\theta)$, for all $f' \in F$ if $f' \in EL_i(f \mid \{\theta''\})$, then $f'\circ\theta^\alpha \in EL_i(f\circ\theta^\alpha \mid \{\theta\})$. This implies that for w, g, $\hat{\theta}$, θ^* j and k given above, Possibility A implies (+)

and Possibility B implies (++) and (+++), where

$$w \notin EL_j(f \mid \{\theta''\}) \text{ for } \theta'' = \theta^{\alpha}(\hat{\theta}) \tag{+}$$

$$g \notin EL_k(f \mid \{\theta''\}) \text{ for } \theta'' = \theta^{\alpha}(\theta^{*}) \tag{++}$$

and for all $i \in N$, for all $\theta' \in \Theta$,

$$f \in EL_i(g \mid \{\theta'\}) \tag{+++}$$

(+) contradicts the fact that $f \in \mathcal{P}^e$ implies ex post individual rationality of f. (++) and (+++) contradict the fact that $f \in \mathcal{P}^e$ implies ex post efficiency of f. Thus Possibilities A and B cannot occur and we conclude that $f \circ \theta^{\alpha} \in \mathcal{P}^e$. ■

Rain

	L	M	R
T	10, 10	10, 5	0, 0
B	5, 5	5, 10	0, 0

Shine

	L	M	R
T	10, 10	5, 5	10, 0
B	5, 5	20, 10	0, 0

Clouds

	L	M	R
T	5, 10	5, 10	10, 0
B	10, 10	5, 5	5, 10

Figure 1

$s_1(\text{Rain}) = T;$

$s_1(\text{Shine}) = T;$

$s_1(\text{Clouds}) = T;$

$s_2(\text{Rain}) = L$

$s_2(\text{Shine}) = L$

$s_2(\text{Clouds}) = L$

$s'_1(\text{Rain}) = T:$

$s'_1(\text{Shine}) = T;$

$s'_1(\text{Clouds}) = T;$

$s'_2(\text{Rain}) = L$

$s'_2(\text{Shine}) = L$

$s'_2(\text{Clouds}) = M$

$s''_1(\text{Rain}) = B;$

$s''_1(\text{Shine}) = B;$

$s''_1(\text{Clouds}) = B;$

$s''_2(\text{Rain}) = M$

$s''_2(\text{Shine}) = M$

$s''_2(\text{Clouds}) = R$

$s^*_1(\text{Rain}) = B;$

$s^*_1(\text{Shine}) = B;$

$s^*_1(\text{Clouds}) = B;$

$s^*_2(\text{Rain}) = M$

$s^*_2(\text{Shine}) = M$

$s^*_2(\text{Clouds}) = L$

Figure 2

Let $m = (\pi_i(i),\ f(i)\ \delta(i))_{i \in N}$

Case 1:

$\forall i \in N$, (i) $\exists \theta \in \Theta$ such that $\theta^*(m) = \{\theta\}$, (ii) $\exists f \in \varphi$ such that $f(i) = f$, and (iii) $\delta(i) = 0$.

$\downarrow$

$\xi(m) = f(\theta)$

Case 2:

(i) $\exists f \in \varphi$ such that $\forall j \in N$, $f(j) = f$, (ii) $\exists i \in N$ such that m_{-i} satisfies Property $\gamma | i$ and (iii) the conditions of Case 1 are not all met.

Case 2A

$K(m) = \{i\}$

$\downarrow$

$\xi(m) = f(\theta_i(m_{-i}))$

Case 2B

Otherwise

$\downarrow$

$\xi(m) = 0$

Figure 3

Case 3:

$\exists i \in N$ such that (i) $\forall j \in N \backslash \{i\}$, $f(i) \neq f(j)$ and (ii) m_{-i} satisfies Property $\gamma | i$.

Case 3A

(i) $K(m) = \{i\}$

(ii) $f(i) \in EL_i(f(j) | \{\theta_1(m_{-1})\})$, $j \neq i$.

$$\downarrow$$

$\xi(m) = f(i)(\theta_1(m_{-1}))$

Case 3B

Otherwise

$$\downarrow$$

$\xi(m) = 0$

Case 4:

Otherwise

Case 4A:

$\exists i \in N$ with $K(m) = \{i\}$

$$\downarrow$$

$(\xi_i(m), \xi_{-i}(m)) = (\Omega, 0)$

Case 4B

Otherwise

$$\downarrow$$

$\xi(m) = 0$

Figure 3 (Contd.)